Objective Measures of Market Efficiency: Applications to Securities Class Actions and Valuations

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ABSTRACT

For every U.S.-listed security for each year between 2001 and 2017, I run four different event studies to calculate four separate objective measures of the market efficiency for that security for that year. These studies provide an objective characterization of that security's market for that year, to determine whether it is sufficiently efficient or not.

I apply these methodologies to Petrobras's American Depositary Receipt (ADR), traded as PBR on the New York Stock Exchange, from 2001 to 2017 and conclude that the *Petrobras Court* reached the incorrect conclusion when it certified a 2010 to 2015 class period because the market for PBR was sufficiently efficient in 2010, 2011, and 2014, but not sufficiently efficient in 2012, 2013, and 2015.

I also apply these methodologies to the valuation of each U.S.-listed firm in 2001-2017. Three examples are as follows: a) the market for GS (Goldman Sachs Group, Inc., common equity) was sufficiently efficient in each year in 2001-2017, and consequently market prices represented value for Goldman Sachs over

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2001-2017; b) the market for MSFT (Microsoft Corporation, common equity) was sufficiently efficient in 2001 and 2003-2017, and therefore market prices represented value for Microsoft Corporation in 2001-2017 except for 2002; and c) the market for AAME (Atlantic American Corporation, common equity) was not sufficiently efficient in any year between 2001-2017, and therefore market prices did not represent value for Atlantic American Corporation in 2001-2017.

Keywords: Market Efficiency; Event Study; Earnings Announcements; Key Developments; Securities Class Actions; Valuation; Tax.

INTRODUCTION

A market is *semi-strong efficient* ("efficient") if prices reflect all publicly available information—see, for example, work by Nobel Prize-winners Eugene Fama[1] and Paul Samuelson,[2] and by other leading scholars.[3] A market is efficient if stock prices adjust very rapidly to new information.[4] Finally, abnormal returns, on average, are zero in an efficient market.[5] To summarize: 1) Prices of securities reflect, *albeit to varying extents*, all publicly available information; 2) prices adjust, *albeit to varying extents*, to new information; and 3) abnormal returns are close to zero, also *albeit to various extents*. Therefore, markets for securities are efficient in varying degrees, referred to as *relative efficiency*.[6]

I run *four different event studies* for every U.S.-listed security from 2001 to 2017 to calculate four different measures of market efficiency based on the frontiers of financial and information economics.[7] I do *not* ascribe any directional component to any potentially material event in this paper, because such an ascription would require subjective judgment and/or additional information.

1. *See, e.g.*, Eugene F. Fama, *Efficient Capital Markets: II*, 46 J. FIN. 1575 (1991); Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, 25 J. FIN. 383 (1970).

2. Paul Samuelson, *An Enjoyable Life Puzzling Over Modern Finance Theory*, 1 ANN. REV. FIN. ECON. 19 (2009).

3. *E.g.*, Burton Malkiel, *Efficient Market Hypothesis*, *in* NEW PALGRAVE DICTIONARY OF MONEY AND FINANCE 127 (John Eatwell, John Milgate, & Peter Newman eds., 1992); Burton Malkiel, *The Efficient Market Hypothesis and Its Critics*, 17 J. ECON PERSP. 59 (2003); LARRY HARRIS, TRADING & EXCHANGES: MARKET MICROSTRUCTURE FOR PRACTITIONERS (2003); ANDREI SHLEIFER, INEFFICIENT MARKETS – AN INTRODUCTION TO BEHAVIORAL FINANCE (2000).

4. Eugene F. Fama, Lawrence Fisher, Michael C. Jensen & Richard Roll, *The Adjustment of Stock Prices to New Information*, 10 INT'L ECON. REV. 1 (1969).

5. *See, e.g.*, Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, *supra* note 1.

6. It is worth emphasizing that a security's being sufficiently efficient or otherwise has no *a priori* bearing on whether it is a good investment or not. *See, e.g.*, JOHN Y. CAMPBELL, ANDREW W. LO & A. CRAIG MACKINLAY, THE ECONOMETRICS OF FINANCIAL MARKETS (1997).

7. For details, see Rajeev Bhattacharya, *Market Efficiency, Trading Volume, and Short Sales Costs & Constraints; A Structural Approach*, SSRN (2018), https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2999356; *see also* Appendix 1 for summary statistics, especially correlation coefficients, of all the four different measures of market efficiency that I use in this paper, for all U.S.-listed securities, for 2001-2017; *see also* Appendix 2 for summary statistics, especially correlation coefficients, of all the four different measures of market efficiency that I use in this paper, for all U.S.-listed common stocks, separately for Nasdaq and non-Nasdaq stocks.

Furthermore, it is *ad hoc* to infer from the description of an event if the event would cause, under market efficiency, the price to go up, down, or stay the same. In other words, it is impossible to objectively determine the market's perceptions immediately prior to any potentially material event and, therefore, to determine whether a particular potentially material event was better-than-expected news, worse-than-expected news, or even news at all.[8] Colloquially speaking, good news or bad news is not the relevant question here. Reaction, overreaction, correction, overcorrection, bounce-back, etc. should be out of the system within a few days after a potentially material event in an efficient market.

The measures that I use in in this paper are completely rules-based, with no judgment, subjectivity, or additional information required of any kind. The only rules are such that a reasonable person can, *a priori*, agree with: a) for the industry index, I use all firms that are in the same major industry sector (same two-digit SIC[9] Code) as the firm in question, and b) abnormal price reaction to an event after a few days is close to zero for an efficient market (as described earlier).

Finally, I rely upon advanced statistical techniques to estimate a relationship between market efficiency and each security for each year, providing an objective characterization of the market for a security in a year to be sufficiently efficient or otherwise.

In this paper, I apply these methodologies to Petrobras's American Depositary Receipt (ADR), trading under the ticker PBR on NYSE, in 2001-2017, and conclude that the *Petrobras Court* was in part right, but in part wrong, when it certified a 2010-2015 class period,[10] because the market for PBR was sufficiently efficient in 2010, 2011, and 2014, but not sufficiently efficient in 2012, 2013, and 2015.

In this paper, I apply these methodologies to value each U.S.-listed firm for each year in 2001-2017. Here are three examples:

1) The market for GS (Goldman Sachs Group, Inc., common equity) was sufficiently efficient in each year in 2001-2017, and therefore, market prices represented value for Goldman Sachs over 2001-2017.
2) The market for MSFT (Microsoft Corporation, common equity) was sufficiently efficient in each of these years except 2002, and therefore, market prices represented value for Microsoft Corporation in these years except 2002. My analyses do not address whether market prices over- or under-estimated value of Microsoft in 2002.
3) The market for AAME (Atlantic American Corporation, common equity) was not sufficiently efficient in any year in 2001-2017, and therefore, market prices did not represent value for Atlantic American Corporation in

8. Paul A. Ferrillo, Fred C. Dunbar & David Tabak, *The "Less Than" Efficient Capital Markets Hypothesis: Requiring More Proof From Plaintiffs in Fraud-On-The-Market Cases*, 78 ST. JOHN'S L. REV. 81 (2004).

9. "Standard Industrial Classification."

10. *In re* Petrobras Sec. Litig., 312 F.R.D. 354 (S.D.N.Y. 2016).

> 2001-2017. My analyses do not address whether market prices over- or under-estimated value of Atlantic American Corporation in any of these years.

Section I describes the four different objective measures of market efficiency and the statistical methodology I use in this paper.[11] Section II details the 2 million Megabytes of data that were relied upon for this paper. Section III outlines the role of market efficiency in securities class actions, and applies the four different objective measures of market efficiency to the recent *Petrobras Securities Litigation*. Section IV discusses the essential importance of market efficiency in valuation, and applies the four different objective measures of market efficiency to the valuation of three firms with publicly listed equity. Section V concludes and provides directions for future research.

I. METHODOLOGY

I use the following *objective* metrics, based on event studies, as *separate negative measures* for the efficiency of the market for a security:[12]

1) Key Developments Abnormal Response (KDAR) of the security, estimated through the Capital Asset Pricing Model (CAPM);
2) KDAR of the security, estimated through the Market Model (MM);
3) Earnings Announcements Abnormal Response (EAAR) of the security, estimated through CAPM; and
4) EAAR of the security, estimated through MM.

Over the past five decades,[13] event studies have become a fundamental part of financial economics, valuation, and litigation. Hundreds of event studies have been conducted in the legal, financial economics, and accounting literatures. Event studies are tests of market efficiency. They test the impact, speed, and unbiasedness of the market's reaction to an event. In an efficient capital market, a security's price reaction to an event is expected to be immediate. Subsequent price movement is expected to be unrelated to the event-period reaction or its

11. *See* Appendix 1 and Appendix 2, *infra*.

12. Bhattacharya, *supra* note 7 ("I use . . . different metrics as separate measures of the efficiency of the market for a stock and describe how they relate to one another. I develop a simple two-equation structural model that shows the impacts on market efficiency of a) short sales costs and constraints and b) trading volume are theoretically indeterminate and are, therefore, empirical questions. I use the panel nature of the data to identify appropriate instruments for the endogenous variables and for the variables that are measured with error. I use Three-Stage Least Squares to estimate this structural model, accounting for the Fama-French Factors (market cap, leverage, book-to-market equity ratio, and price-to-earnings ratio) and institutional ownership. I analyze Nasdaq stocks and non-Nasdaq stocks separately,

> 1) *Ceteris paribus*, the impact of short sales costs and constraints of a stock on the efficiency of the market for the stock is *not* significantly negative.
> 2) *Ceteris paribus*, the impact of normalized trading volume of a stock on the efficiency of the market for the stock is *not* significantly positive.").

13. *See* Fama *et al.*, *supra* note 4; *see also* Ray Ball & Phillip Brown, *An Empirical Evaluation of Accounting Income Numbers*, 6 J. ACCT. RES. 159 (1968); *see also* Ray Ball, Eugene F. Fama, Lawrence Fisher, Michael C. Jensen, and Richard Roll, *Retrospective Comments*, *in* THE FAMA PORTFOLIO (John Cochrane & Tobias Moskowitz, eds., 2017).

prior period return.[14] Event studies serve an important purpose in capital market research as a way of testing market efficiency, because systematically nonzero abnormal security returns that persist after a particular type of corporate event are inconsistent with market efficiency.[15]

I do *not* ascribe any directional component to any potentially material event[16] in this paper. It requires subjective judgment to infer from the description of an event if market efficiency would require the price of the security to go up, down, or stay the same. In other words, it is impossible to objectively determine the market's perceptions immediately prior to any potentially material event. To determine whether a particular potentially material event was better information than expected, worse information than expected, or just no surprise at all is not possible without additional information or *ad hoc* judgment.[17] Colloquially speaking, good news or bad news is not the relevant question here; and reaction, overreaction, correction, overcorrection, bounce-back, etc. should, in an efficient market, be out of the system within a few days after a potentially material event. See, for instance, Judge Jed Rakoff's discussion of directionality, and his criticisms of the subjective and *ad hoc* marking of directionality of events in the dueling expert reports, in his judgment in *Petrobras Securities Litigation.*[18]

How to identify a potentially material event leads to the following two paths of study:

- For the first path of event studies research, I rely upon the marking of an event as a Key Development by Capital IQ, a service of Standard & Poor's, to tag an event as potentially material.[19,20]
- For the second path of event studies research, I designate each earnings announcement as a potentially material event. Such a study of earnings

14. S.P. Kothari, *Capital Markets Research in Accounting*, 31 J. ECON. & ACCT. 1 (2001).

15. S.P. Kothari & Jerold B. Warner, *Econometrics of Event Studies*, *in* HANDBOOK OF EMPIRICAL CORPORATE FINANCE (Espen Eckbo, ed., 2007).

16. If the event time is at or after 4 PM, I consider the next day as the effective day.

17. Ferrillo *et al*., *supra* note 8.

18. For instance, an expert used the presence or absence of the text corrupt in the description of an event to determine the relevant event's direction. *In re* Petrobras Sec. Litig., 312 F.R.D. 354 (S.D.N.Y. 2016).

19. For each security and for each year, I calculate the average, for that security and that year, of the absolute value of the cumulative abnormal return for the security over the last three days of a week's window following each Key Development for that security in that year—I call this the *Key Developments Abnormal Response (KDAR) for that security for that year and use it as a negative measure for the efficiency of the market for that security in that year.*

20. As described earlier, I do not try to ascribe any directional component to any Key Development identified by Capital IQ—it is impossible to objectively determine whether a particular Key Development was better than expected news, worse than expected news, or simply no surprise at all.

announcements' impact on security prices has a long and distinguished history.[21, 22, 23]

For each security, I estimate the normal (and implicitly the abnormal) daily return for each year, and more specifically for each trading day in that year, by using the Capital Asset Pricing Model and Market Model separately.[24]

- I rely upon the Capital Asset Pricing Model (CAPM)[25], to conduct the calculations set forth above for each security.[26]

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21. *See, e.g.*, Ball & Brown, *supra* note 13; Daniel W. Collins & S.P. Kothari, *An Analysis of Intertemporal and Cross-Sectional Determinants of Earnings Response Coefficients*, 11 J. ACCT. & ECON. 143 (1989).

22. I calculate the average for each security for that year, of the absolute value of the cumulative abnormal return for the security over the last three days of a week's window following each earnings announcement for that security in that year—I call this the *Earnings Announcements Abnormal Response (EAAR) for that security for that year and use it as a negative measure for the efficiency of the market for the security in that year.*

23. Any estimate of the market's consensus prediction of the EPS at the point of an EPS announcement, whether it is by using mean/medians of analyst forecasts, or by using valuation models, is sensitive to the methodology used to estimate the market's perception and the deviation from it. The actual announced EPS or its deviation from consensus does not enter into my calculations in this paper. For detailed descriptions of biases and robustness in the calculation of deviation from consensus, *see*, *e.g.*, Chin-Han Chiang, Wei Dai, Jianqing Fan, Harrison Hong & Jun Tu, *Robust Measures of Earnings Surprises*, J. FIN. (forthcoming 2016). For a comprehensive survey of the literature on quality, bias, and predictability of earnings forecasts, see S. P. Kothari, Eric So & Rodrigo Verdi, *Analysts' Forecasts and Asset Pricing: A Survey*, 8 ANN. REV. FIN. ECON. 197 (2016); the impact of analyst incentives on analyst forecasts is beyond the scope of this paper. For an example of recent research, see Rajeev Bhattacharya, Andrew Hartnett, Michael McDonald, Rebecca Nelson & Stoyan Stoyanov, *Analyst Forecasts and Analyst Incentives*, SSRN (2017). For reasons explained *supra*, it is impossible to objectively distill how much information about an actual earnings announcement leaked out before the actual announcement; i.e., to *objectively* distill the true surprise component of an earnings announcement is impossible.

24. For each potentially material event—determined as below—I calculate the cumulative abnormal return for the security over the last three days of a week's window following the potentially material event. For each security, for each potentially material event at time *t*, I calculate the cumulative abnormal return for the *[(T – 1 week) ≤ t ≤ (T + 1 week)]* window, thus allowing for the possibility that material information could leak out about a week before the potentially material event and that it takes up to a week after the potentially material event for the unanticipated information to have its impact on security returns. From this cumulative abnormal return, I subtract the cumulative abnormal return for the *[(T – 1 week) ≤ t ≤ (T + 4 days)]* window in order to *calculate the cumulative abnormal return for the security over the last three days of a week's window following the potentially material event for that security*—this, as discussed above, is closer to zero as the closer a market is to efficiency.

25. ZVI BODIE, ALEX KANE & ALAN MARCUS, INVESTMENTS (2008), describe CAPM as a set of predictions concerning equilibrium expected return on risky assets. Harry Markowitz laid down the foundations of modern portfolio management in 1952. The CAPM was developed 12 years later in articles by William Sharpe, John Lintner and Jan Mossin. For a non-technical survey of CAPM, see Rajeev Bhattacharya, *CAPM*, *in* PALGRAVE ENCYCLOPEDIA OF STRATEGIC MANAGEMENT (Mie Augier & David Teece, eds., 2016). For a study of more sophisticated versions of CAPM from a behavioral perspective, see *HERSH SHEFRIN,* A BEHAVIORAL APPROACH TO ASSET PRICING (2008*);* Hersh Shefrin, *Building on Kahneman's Insights in the Development of Behavioral Finance*, 44 LOY. UNIV. CHI. L.J. 1401 (2013); and Valerio Potì & Hersh Shefrin, *The Signature of Sentiment in Conditional Consumption CAPM Estimates: A Note*, 2 J. BEHAV. & EXPERIMENTAL FIN. 1 *(2014).* The formula for CAPM is as follows:

$$Rp_{i,t} = \beta_{M,i} Rp_{M,t} + AbnR_{i,t}$$

where:

$Rp_{i,t} = R_{i,t} - R_{f,t}$ = risk premium on day *t* of security *i*

$Rp_{M,t} = R_{M,t} - R_{f,t}$ = risk premium on day *t* of the market index

$AbnR_{i,t}$ = abnormal return on day *t* of security *i*.

26. For the baseline model, I use daily returns on three-month T-bills as the risk-free rate, the daily returns, including all distributions, on the value-weighted market portfolio as the market return – VWRETD indices contain . . . daily . . . returns, including all distributions, on a value-weighted market

- Separately and additionally, I also rely upon the Market Model (MM),27 to run the same calculations for each security for each year and day of the year.

As described earlier, I performed four different event studies for every U.S.-listed security available between 2001 and 2017. From these event studies, I calculated four different objective negative measures of the efficiency of the market for each of the assets for every year of the range. For each calendar year, I then transformed these negative measures of market efficiency for each stock into a positive measure of market efficiency in two ways: 1) By calculating the natural exponent of the negative of the measure, and 2) By calculating the negative of the measure. I then controlled for each measure type (as described above) and found a positive measure (as described above), and the interaction of each security and each year (the coefficient of this interaction term is the security-year-specific impact).[28] Subsequently, for each security and each year, I test this security-year-specific impact using one-tailed hypothesis testing,[29] at a 5% level of significance. This essentially provides an objective characterization of the market for a security in a year to be sufficiently efficient or not.

II. DESCRIPTION OF DATA

1) Security-days that had positive closing price, positive shares outstanding[30], and other restrictions[31] from CRSP:

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portfolio (excluding American Depository Receipts) (CRSP information) – and I also check for robustness: a) using returns on four-week, six-month, and one-year T-bill rates separately as risk-free rates, and b) returns on the equally-weighted market portfolio – EWRETD indices contain . . . daily . . . returns, including all distributions, on an equally-weighted market portfolio (including ADRs) (CRSP information) – and on the S&P Composite Index as the market return.

27. *See, e.g.* CAMPBELL ET AL., *supra* note 6. The Market Model is similar to the much-studied Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT). The formula for the Market Model is as follows:

$$R_{i,t} = \alpha + \theta_{f,i}R_{f,t} + \theta_{M,i}R_{M,t} + \theta_{C,i}R_{C,i,t} + AbnR_{i,t}$$

where:

$R_{i,t}$ = return on day t of security i

$R_{f,t}$ = risk-free return on day t

$R_{M,t}$ = return on day t of the market index

$R_{C,i,t}$ = return on day t of comparables to firm i: I use the value-weighted average daily returns of equities of all firms in the same major industrial sector as firm i – i.e., with the same 2-digit Standard Industrial Classification (SIC) Code – as the corresponding return for comparables

$AbnR_{i,t}$ = abnormal return on day t of security i.

28. First, I regress each market efficiency measure by using a fixed effect for each measure type and each transform and calculate the residuals. Second, I regress these residuals by using a fixed effect for each security and each year.

29. For a discussion of one-tailed hypothesis testing, see Bhattacharya, *supra* note 7.

30. For a small number of stock-days, I found multiple entries when sorted by ticker, CUSIP, and date – for these stock-days, I calculate weighted averages using trading volume as weights.

31. For a security to be in my sample for a particular calendar year, I require that the security have tra ded for at least half the trading days in that calendar year. Also, if ret notin (-44.0,-55.0,-66.0,-77.0,-88.0,-99.0); if dlret notin (-55.0,-66.0,-88.0,-99.0); if dlretx notin (-55.0,-66.0,-88.0,-99.0); if dclrdt ne 0; if trtscd ne 0; if nmsind ne 0; if vol ne -99.0; if prc > 0; if bid > 0; if ask > 0; if vol > 0; if shrout > 0; if ticker ne "?"; if ticker ne ""; if cusip ne ""; if substr(ticker,1,4) ne "ZZZZ"; if substr(cusip,7,1) in

a) date
b) closing price
c) (discrete) return
d) shares outstanding
e) trading volume
f) closing bid
g) closing ask
h) exchange membership
i) number of market makers
j) SIC Code
k) value-weighted market return of all publicly traded U.S. securities[32]
l) equally-weighted market return of all publicly traded U.S. securities[33]

2) And had data on the following variables:
a) Earnings announcements (from I/B/E/S)
b) Key Developments (from Capital IQ)
c) Compustat-CRSP Merged Database[34]

III. SECURITIES CLASS ACTIONS

Market efficiency is discussed at the class certification stage of securities class actions, where the court attempts to determine if the plaintiffs' claims are best tried individually or collectively, which allows numerous plaintiffs to pursue essentially the same claim against the defendant at the same time. At the class certification stage, plaintiffs can present evidence that they traded shares in an efficient market. Defendants can rebut the presumption of reliance by challenging actual reliance or market efficiency.[35]

One significant decision by the U.S. District Court for the District of New Jersey enumerated several factors that should be considered, including: (1) the average weekly trading volume; (2) the number of security analysts following and reporting on the security; (3) the extent to which market makers traded the security; (4) the issuer's eligibility to file a U.S. Securities and Exchange Commission registration Form S-3; and (5) the cause-and-effect relationship between material disclosures and changes in the security's price.[36] These

("0","1","2","3","4","5","6","7","8","9"); if substr(cusip,8,1) in ("0","1","2","3","4","5","6","7","8","9").

32. VWRETD indices contain the daily returns, including all distributions, on a value-weighted market portfolio (excluding American Depository Receipts (ADRs)). (CRSP information)

33. EWRETD indices contain . . . daily . . . returns, including all distributions, on an equally-weighted market portfolio (including ADRs). (CRSP information)

34. Compustat (including Capital IQ) data sources rely upon the S&P identification of GVKEY. In this paper, I rely upon the mapping of GVKEY to TICKER used by CRSP to match its data to Compustat. In order to maintain the integrity of the linkages between the different sources of data I rely upon, I treat a missing observation as a missing observation, not as a zero.

35. *Basic v. Levinson,* 485 U.S. 224 (1988).

36. *Cammer v. Bloom*, 711 F. Supp. 1264, 1286-87 (D.N.J. 1989).

Cammer Factors have been adopted by a number of other courts.[37] Still other courts have added additional considerations. For instance, one court considered the company's market capitalization and the size of the public float for the security,[38] while another considered the ability to sell short the security and the level of autocorrelation between the security's prices.[39]

In this paper, I emphasize relative efficiency that: 1) prices of securities reflect, *albeit to varying extents*, all publicly available information: 2) prices adjust, *albeit to varying extents*, to new information; and 3) abnormal returns are close to zero, also *albeit to various extents*. Therefore, markets for securities are efficient to varying degrees.[40, 41]

In order to appreciate how trading volume impacts market efficiency, it is necessary to understand why a trade occurs. In particular, investors trade among themselves due to differences between them,[42] and volume reflects a lack of consensus regarding price.[43] However, there exists no reason for higher dispersion in investor valuations to lead to higher market efficiency and therefore, the impact on market efficiency of normalized trading volume everything else remaining the same, is fundamentally an empirical question, to which the empirical answer is that the efficiency of the market for a stock is *not* significantly and positively affected by trading volume.[44] The demand for market making services is an increasing function of trading volume, for instance, through higher dispersion of the valuation profile. As a corollary, everything else remaining the same,[45] a firm will more likely enter (or not exit) the market for market making services due to higher trading volume (and thus, higher market

37. *See In re* DVI, Inc. Sec. Litig., 639 F.3d 623, 633 n.14; Teamsters Local 445 Freight Div. Pension Fund v. Bombardier, Inc., 546 F.3d 196, 204 n. 11 (2d Cir. 2008); *In re* Xcelera.com Sec. Litig., 430 F.3d 503, 508 (1st Cir. 2005); Unger v. Amedisys Inc., 401 F.3d 316, 323 (5th Cir. 2005); Gariety v. Grant Thornton, LLP, 368 F.3d 356, 368 (4th Cir. 2004); Binder v. Gillespie, 184 F. 3d 1059, 1064-65 (9th Cir. 1999).

38. *See* Krogman v. Sterritt, 202 F.R.D. 467, 478 (N.D. Tex. 2001).

39. *See* In re Polymedica Corp. Sec. Litig., 432 F.3d 1, 18 n. 21 (1st Cir. 2005).

40. *See, e.g.,* Halliburton Co. v. Erica P. John Fund, Inc. (Halliburton II), 573 U.S. 258, 357 (2014), and *In re* Petrobras Sec. Litig., 312 F.R.D. 354 (S.D.N.Y. 2016).

41. For more details on the legal implications of market efficiency for securities class actions, see Rajeev Bhattacharya & Stephen O'Brien, *Arbitrage Risk and Market Efficiency—Applications to Securities Class Actions*, 55 SANTA CLARA L. REV. 643 (2015).

42. Jiang Wang, *A Model of Competitive Stock Trading Volume*, 102 J. POL. ECON. 127 (1994). *See also* Jonathan Karpoff, *The Relation Between Price Changes and Trading Volume: A Survey*, 22 J. FIN & QUANTITATIVE ANALYSIS 109 (1987); MAUREEN O'HARA, MARKET MICROSTRUCTURE THEORY (1997); Scott Stickel & Robert Verrecchia, *Evidence that Trading Volume Sustains Stock Price Changes*, 50 FIN. ANALYSTS J. 57 (1994); Lawrence Blume, David Easley, & Maureen O'Hara, *Market Statistics and Technical Analysis: The Role of Volume*, 49 J. FIN. 153 (1994); and Edie Hotchkiss, Michael Goldstein, & Erik Sirri, *Transparency and Liquidity: A Controlled Experiment on Corporate Bonds*, 20 R. FIN. STUD. 235 (2007).

43. William Beaver, *The Information Content of Annual Earnings Announcements*, 6 J. ACCT. RES. 67 (1968).

44. Bhattacharya, *supra* note 7; Bhattacharya & O'Brien, *supra* note 41.

45. In particular, keeping constant other incentives of investment banks, such as profits from proprietary trading.

making profits). However, the greater the number of market makers, the more likely competition for trades places downward pressure on the transaction costs. Economies of scale will determine the equilibrium impact on the price of market making services.[46] The directional impact of the number of market makers for a security on market efficiency for that security can only be determined empirically: The number of market makers does not significantly and positively affect the market efficiency.[47] Recent empirical work also shows that short sales costs and constraints do not negatively impact market efficiency.[48]

As described earlier, I perform four different event studies for every U.S.-listed security from 2001 to 2017 and calculate four different objective measures of market efficiency for an asset each year.[49] I transform each calendar year's stock's negative market efficiency into a positive market efficiency measure in two ways: 1) by calculating the natural exponent of the negative of the measure, and 2) by calculating the negative of the measure. I then control for each measure type (as described above), each transformation into a positive measure (as described above), and then estimate each security-year-specific impact. For each security and each year, I test this security-year-specific impact, using one-tailed hypothesis testing, at a 5% level of significance, which objectively characterizes the market for a security in a year to be sufficiently efficient or otherwise.

In this paper, I apply these methodologies to Petrobras's ADR (PBR) on NYSE, in 2001-2017. Ultimately, I conclude that the *Petrobras* Court was partially right and partially wrong when it certified a 2010-2015 class period[50] because the market for PBR was sufficiently efficient in 2010, 2011, and 2014, but not sufficiently efficient in 2012, 2013, and 2015.

IV. VALUATION

As discussed above, if and only if the market for an asset is *sufficiently* efficient, and conditional on public information, the asset's market price remains a reasonable estimate of value. Whether market price of a particular asset (or portfolio of assets) serves as a reasonable approximation of that particular asset's value depends solely on whether the market for that particular asset was *sufficiently* efficient. In this paper, I use four different objective measures of the market efficiency for an asset (detailed in Section Objective Measures of Market Efficiency).[51] Relying upon advanced statistical techniques, I also estimate a relationship between market efficiency and each security for each year, which

46. *See*, *e.g.*, Rajeev Bhattacharya, *Non-Monotonicity of Equilibrium Price*, SSRN (2017), https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2980215.

47. Bhattacharya & O'Brien, *supra* note 41.

48. *See* Bhattacharya, *supra* note 7.

49. *See* Appendix 1 and Appendix 2, *infra*.

50. *In re* Petrobras Sec. Litig., 312 F.R.D. 354 (S.D.N.Y. 2016).

51. *See* Bhattacharya, *supra* note 7. *See* Appendix 1 and Appendix 2, *infra*.

provides an objective market character for a security in a sufficiently efficient year. If a valuer proposes using market price(s) as approximation for value(s), such a proposal will need to be supported through sufficiently high measures of the market(s) efficiency for the relevant assets. Conversely, an argument against using market prices as value in a particular context will be meaningful if, and only if, objective measures of market(s) efficiency for the relevant assets show the corresponding markets to not be sufficiently efficient.

In this paper, I apply these methodologies to each U.S.-listed firm for each year in 2001-2017; here are three examples:

1) The market for GS (Goldman Sachs Group, Inc., common equity) was sufficiently efficient in each year in 2001-2017, and therefore, the market prices represented value.
2) The market for MSFT (Microsoft Corporation, common equity) was sufficiently efficient in each of these years except 2002, and therefore, market prices represented value for Microsoft Corporation in these years except 2002. My 2002 Microsoft analyses do not address whether market prices over- or under-estimated value.
3) The market for AAME (Atlantic American Corporation, common equity) was not sufficiently efficient in any year in 2001-2017, and therefore, market prices did not represent value. My analyses do not address whether market prices over- or under-estimated value of Atlantic American Corporation in any of these years.

Mark-to-Market (MTM or M2M) accounting[52] is based on the premise that the measured assets consist only of components that are part of, or similar to, assets that are trading in sufficiently efficient markets. Like above, an objective measure of market efficient for the relevant assets showing the market to be sufficiently efficient must support any proposal to use MTM. Conversely, an argument against MTM in a particular context will be meaningful if and only if market efficiency's objective measures show the corresponding markets to not be sufficiently efficient.

A comparables approach to valuation is essentially a joint test of the market efficiency for the firm's (or an asset, or a portfolio) equity value.[53] For example, by using Google and Apple stocks as comparables to Facebook and relying upon Price/Earnings (P/E) Ratio as the relevant metric, we can test the null hypothesis

52. *See* Franklin Allen & Elena Carletti, *Mark-to-Market Accounting and Liquidity Pricing*, 45 J. ACCT. & ECON. 358 (2008), for a thoughtful, albeit somewhat dated, summary of the pros and cons of this approach. *See* Andrew Ellul, Chotibhak Jotikasthira, Christian Lundblad & Yihui Wang, *Mark-To-Market Accounting and Systemic Risk: Evidence from the Insurance Industry*, 29 ECON. POL'Y 297 (2014), for a post-Global Financial Crisis study.

53. A comparable to a firm is another firm that shares similar characteristics. ASWATH DAMODARAN, INVESTMENT VALUATION (2012) ("The value of most assets, from the house you buy to the stocks you invest in, are based on how similar assets are priced in the marketplace . . . the value of an asset is derived from the pricing of comparable assets, standardized using a common variable such as earnings, cash flows, book value, or revenues.") This applies to all valuations that are based on comparables—real estate, for instance, strongly relies upon comps-based valuation methods.

that Facebook is correctly valued (i.e., the market for Facebook common stock is semi-strong form efficient).

Any conclusion based on the differences between the P/E Ratios of Facebook on one hand, and those of Google and Apple on the other, remains grounded on the implicit assumption that Google and Apple common stocks are trading on sufficiently efficient markets. It is possible for the stock market to reject this particular null hypothesis because the stock market misprices Facebook, and/or because Google and/or Apple stocks trade on inefficient markets. Therefore, in valuation contexts, whether the comparables approach to valuation is meaningful depends on the market's efficiency for a given asset's comparables. As stated above, objective measures of the market efficiency for the comparables must support any proposal to use the comparables valuation approach. Conversely, any argument against using the comparables' market prices will be valid if and only if objective measures of market efficiency show the corresponding markets are not sufficiently efficient.

The extent to which the market for a security is efficient informs whether an investor or analyst needs to acquire additional information about the firm issuing the security. If a security trades in a *sufficiently* efficient market, traded price is a good estimate of value per share, conditional on public information; otherwise, additional information will have to be acquired by the investor or analyst at additional cost. It is worth emphasizing, however, that a security being sufficiently efficient or otherwise has no *a priori* bearing on whether it is a good investment or not.

V. CONCLUSIONS AND FUTURE RESEARCH

In this paper, I utilized the following metrics as separate and objective negative measures, based on event studies, of the market's efficiency for a security:[54]

- Key Developments Abnormal Response (KDAR) of the security, estimated through the Capital Asset Pricing Model (CAPM)
- KDAR of the security, estimated through the Market Model (MM)
- Earnings Announcements Abnormal Response (EAAR) of the security, estimated through CAPM
- EAAR of the security, estimated through MM

I did *not* ascribe any directional component to any potentially material event in this paper, since it requires subjective judgment and/or additional information, and it is *ad hoc* to infer from the description of an event if the event would cause, under market efficiency, the price to go up, down, or stay the same. It is impossible to objectively determine the market's perceptions immediately prior to any potentially material event, and therefore, to determine whether a particular potentially material event was better-than-expected news, worse-than-expected

54. *See* Appendix 1 and Appendix 2, *infra*.

news, or even news at all. Colloquially speaking, good news or bad news is not the relevant question here. Reaction, overreaction, correction, overcorrection, bounce-back, etc. should be out of the system within a few days after a potentially material event in an efficient market.

These measures are completely rules-based, with no judgment, subjectivity, or additional information required of any kind. The only rules are such that every reasonable person can, *a priori*, agree on: a) for the industry index, I used all firms that are in the same major industry sector (same two-digit SIC Code) as the firm in question, and b) abnormal price reaction to an event after a few days is close to zero for an efficient market.

Finally, I relied upon advanced statistical techniques[55] to estimate a relationship between market efficiency and each security for each year, which provided an objective characterization of the market for a security in a year to be sufficiently efficient or otherwise. I applied these methodologies to Petrobras's ADR (PBR, on NYSE) in 2001-2017, and concluded that the *Petrobras* Court got it partially right and partially wrong when it certified a 2010-2015 class period,[56] because the market for PBR was sufficiently efficient in 2010, 2011, and 2014, but not sufficiently efficient in 2012, 2013, and 2015.

In this paper, I applied these methodologies to value each U.S.-listed firm for each year in 2001-2017; here are three examples:

1) The market for GS (Goldman Sachs Group, Inc., common equity) was sufficiently efficient in each year in 2001-2017, and therefore, market prices represented value for Goldman Sachs over 2001-2017.
2) The market for MSFT (Microsoft Corporation, common equity) was sufficiently efficient in each of these years except 2002, and therefore, market prices represented value for Microsoft Corporation in these years except 2002. My analyses do not address whether market prices over- or under-estimated value of Microsoft in 2002.
3) The market for AAME (Atlantic American Corporation, common equity) was not sufficiently efficient in any year in 2001-2017, and therefore, market prices did not represent value for Atlantic American Corporation in 2001-2017. My analyses do not address whether market prices over- or under-estimated value of Atlantic American Corporation in any of these years.

Future research projects include:

1) Application of the measures studied in Rajeev Bhattacharya, "An Option Theoretic Approach to Market Efficiency," SSRN, 2018, which is a working paper that analyzes five measures of a stock's market efficiency that depend on the deviation from put-call parity of options on the stock.
    a) Based on implied volatility spread, following Martijn Cremers and David Weinbaum, "Deviations from Put-Call Parity and Stock Return Predictability," Journal of Finance and Quantitative Analysis, 2010,

55. First, I regress each market efficiency measure by using a fixed effect for each measure type and for each transform, and calculate the residuals. Second, I regress these residuals by using a fixed effect for each security and each year.

56. *In re* Petrobras Sec. Litig., 312 F.R.D. 354 (S.D.N.Y. 2016).

Kaushik Amin, Joshua Coval and Nejat Seyhun, "Index Option Prices and Stock Market Momentum," The Journal of Business, 2004, and Stephen Figlewski and Gwendolyn Webb, "Options, Short Sales, and Market Completeness," Journal of Finance, 1993.

b) Relative price spread, following Eli Ofek, Matthew Richardson and Robert F. Whitelaw, "Limited arbitrage and short sales restrictions: evidence from the options markets," Journal of Financial Economics, 2004, Robert Merton, "The Relationship Between Put and Call Option Prices: Comment," Journal of Finance, 1973, and Hans Stoll, "The Relationship Between Put and Call Option Prices," Journal of Finance, 1969.

2) Application of the theoretical and econometric methodologies of this paper and Rajeev Bhattacharya, "Market Efficiency, Short Sales Costs & Constraints, and Trading Volume; A Structural Approach," SSRN, 2018 to the three measures of market efficiency, based on securities prices following random walks in efficient markets, used by Charles Cao, Bing Liang, Andrew Lo and Lubomir Petrasek, "Hedge Fund Holdings and Stock Market Efficiency," Review of Asset Pricing Studies, 2018.

3) Application of the theoretical and econometric methodologies of this paper and Rajeev Bhattacharya, "Market Efficiency, Short Sales Costs & Constraints, and Trading Volume; A Structural Approach," SSRN, 2018 to the two measures of market efficiency, based on the asymmetry between positive and negative market returns, used by Arturo Bris, William Goetzmann and Ning Zhu, "Efficiency and the Bear: Short Sales and Markets Around the World," Journal of Finance, 2007. Use of other alternative measures for market efficiency, such as intraday bid-ask spreads—such analyses are only possible now with Big Data storage, processing power, and techniques.

4) Structural model studying the relation between capital markets and industrial organization of corresponding product markets.

5) The framework of an expected utility-maximizing investor, using different utility functions.

APPENDIX 1

Summary Statistics
All U.S.-Listed Securities
2001-2017

Summary Statistic	Measure	KDAR (CAPM)	KDAR (MM)	EAAR (CAPM)	EAAR (MM)
MEAN		0.01210	0.01190	0.02944	0.02868
STD		0.01562	0.01562	0.03733	0.03708
N		81,340	81,340	73,098	73,098
CORR	**KDAR (CAPM)**	1.00000	0.98225	0.37905	0.38507
CORR	**KDAR (MM)**	0.98225	1.00000	0.38128	0.40277
CORR	**EAAR (CAPM)**	0.37905	0.38128	1.00000	0.97299
CORR	**EAAR (MM)**	0.38507	0.40277	0.97299	1.00000

APPENDIX 2

Summary Statistics
All U.S.-Listed Equities
2001-2017

Summary Statistic	Measure	KDAR (CAPM)	KDAR (MM)	EAAR (CAPM)	EAAR (MM)
MEAN		0.01313	0.01296	0.02858	0.02792
STD		0.01629	0.01628	0.03450	0.03412
N		55,866	55,866	47,020	47,020
CORR	**KDAR (CAPM)**	1.00000	0.98426	0.36738	0.37382
CORR	**KDAR (MM)**	0.98426	1.00000	0.37081	0.39126
CORR	**EAAR (CAPM)**	0.36738	0.37081	1.00000	0.97207
CORR	**EAAR (MM)**	0.37382	0.39126	0.97207	1.00000

APPENDIX 2

Summary Statistics
All Nasdaq Equities
2001-2017

Summary Statistic	Measure	KDAR (CAPM)	KDAR (MM)	EAAR (CAPM)	EAAR (MM)
MEAN		0.01528	0.01521	0.03271	0.03229
STD		0.01819	0.01820	0.03877	0.03829
N		34,544	34,544	27,657	27,657
CORR	**KDAR (CAPM)**	1.00000	0.98749	0.34723	0.35313
CORR	**KDAR (MM)**	0.98749	1.00000	0.35194	0.36699
CORR	**EAAR (CAPM)**	0.34723	0.35194	1.00000	0.97772
CORR	**EAAR (MM)**	0.35313	0.36699	0.97772	1.00000

APPENDIX 2

Summary Statistics
All Non-Nasdaq Equities
2001-2017

Summary Statistic	Measure	KDAR (CAPM)	KDAR (MM)	EAAR (CAPM)	EAAR (MM)
MEAN		0.00964	0.00930	0.02269	0.02167
STD		0.01180	0.01167	0.02616	0.02580
N		21,322	21,322	19,363	19,363
CORR	**KDAR (CAPM)**	1.00000	0.96954	0.36836	0.37160
CORR	**KDAR (MM)**	0.96954	1.00000	0.36472	0.40117
CORR	**EAAR (CAPM)**	0.36836	0.36472	1.00000	0.95194
CORR	**EAAR (MM)**	0.37160	0.40117	0.95194	1.00000